

# Workshop on Nonlinear Partial Differential Equations VIII

On Sep. 26. 2020

## 1. Workshop Information

### Announcement:

In order to enhance the communications among the mathematicians on the subject of partial differential equations, geometric analysis and related topics, we plan to hold “min-workshop on Geometry PDEs” on Sep. 26 2020. We will invite some experts to share ideas and results on recent research, and discuss current challenging issues.

### Organizing committee:

Mijia Lai, Shanghai Jiao Tong University  
Congming Li, Shanghai Jiao Tong University  
Fang Wang, Shanghai Jiao Tong University,  
Chunqin Zhou, Shanghai Jiao Tong University  
Ran Zhuo, Shanghai Jiao Tong University

### Venues:

Room 813, No. 6 Building, Science Buildings  
Minhang Campus  
Shanghai Jiao Tong University  
800 Dongchuan Road

### Hotel:

Redding Mann Hotel  
Address: No.178 Yongping Nan Road, Minhang District, Shanghai  
200240, China  
Telephone: 021-33882999

### Contact Us

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## 2. Schedule

Sep. 26, Saturday: Room 813, No. 6 Building, Science Buildings	
Forenoon	
8: 20-8: 30	Opening ceremony
<b>Chair: Congming Li</b>	
8: 30-9: 30	Speaker: Qianqiao Guo Title: Subcritical approach to sharp Hardy-Littlewood-Sobolev type inequalities on the upper half space
9: 30-10: 30	Speaker: Genggeng Huang Title: Analyticity of the solutions of degenerate fully non-linear elliptic equations
10: 30-11: 00	Tea break
11: 00-12: 00	Speaker: Yutian Lei Title: A Liouville theorem for an integral equation of GL type
12: 00-14: 30	Lunch
Afternoon	
<b>Chair: Qianqiao Guo</b>	
14: 30-15: 20	Speaker: Yuanyuan Lian Title: 完全非线性椭圆方程的边界 $C^{1,\alpha}$ 正则性
15: 20-15: 50	Tea break
15: 50-16: 40	Speaker: Zhenjie Li Title: Symmetry of solutions to semilinear elliptic equations via Morse index
16: 40-17: 30	Speaker: Ran Zhuo Title: Classification of anti-symmetric solutions to nonlinear fractional Laplace equations
17: 30-18: 00	Free discussion

### 3. Titles and Abstracts

#### Subcritical approach to sharp Hardy-Littlewood-Sobolev type inequalities on the upper half space

Speaker: Qianqiao Guo (Northwestern Polytechnic University)

We establish the reversed sharp Hardy-Littlewood-Sobolev (HLS for short) inequality on the upper half space and obtain a new HLS type integral inequality on the upper half space (extending an inequality found by Hang, Wang and Yan) by introducing a uniform approach. The extremal functions are classified via the method of moving spheres, and the best constants are computed. The new approach can also be applied to obtain the classical HLS inequality and other similar inequalities. This is a joint work with Prof. Jingbo Dou and Prof. Meijun Zhu.

#### Analyticity of the solutions of degenerate fully non-linear elliptic equations

Speaker: Genggeng Huang (Fudan University)

In this talk, we mainly focus on the analyticity of the solutions. We first review some known results to the authors on the analyticity of solutions. Then we report the idea of Kato in the proof of the analyticity. Finally, we give the application of Kato's idea in the proof for the analyticity of the solutions in fully non-linear degenerate elliptic equations.

## A Liouville theorem for an integral equation of GL type

Speaker: Yutian Lei (Nanjing Normal University)

In this talk, we will introduce a Liouville-type result of the non-linear integral equation

$$u(x) = \vec{l} + C_* \int_{\mathbb{R}^n} \frac{u(1 - |u|^2)}{|x - y|^{n-\alpha}} dy. \quad (1)$$

Here  $u : \mathbb{R}^n \rightarrow \mathbb{R}^k$  is a bounded, uniformly continuous and differentiable function with  $k \geq 1$  and  $1 < \alpha < n$ ,  $\vec{l} \in \mathbb{R}^k$  is a constant vector, and  $C_*$  is a real constant. If  $u$  is the finite energy solution, we will prove that  $|\vec{l}| \in \{0, 1\}$ . Furthermore, we also give a Liouville type theorem (i.e.,  $u \equiv \vec{l}$ ).

## 完全非线性椭圆方程的边界 $C^{1,\alpha}$ 正则性

Speaker: Yuanyuan Lian (Shanghai Jiao Tong University)

我们研究了逐点的边界  $C^{1,\alpha}$  正则性, 得到如下结果: 若边界  $\partial \Omega$  在  $x_0$  点是  $C^{1,\alpha}$  的, 则解在  $x_0$  点是  $C^{1,\alpha}$  的。这一结果对 Laplace 方程都是新的, 且证明方法简单。

## Symmetry of solutions to semilinear elliptic equations via Morse index

Speaker: Zhenjie Li (Shanghai Jiao Tong University)

In this talk, I will show symmetry results for solutions of elliptic equations in a ball or in an annulus in  $\square^n$ ,  $n \geq 2$ , in the case where the nonlinearity has a convex first derivative. More precisely, I show that solutions having Morse index  $j \leq n$  are foliated Schwarz

symmetric, i.e. they are axially symmetric with respect to an axis passing through the origin and nonincreasing in the polar angle from this axis.

(from paper: F. Pacella and T. Weth, Proceedings of the American Mathematical Society, 135,(2007), 1753-1762)

## Classification of anti-symmetric solutions to nonlinear fractional Laplace equations

Speaker: Ran Zhuo (Shanghai Jiao Tong University)

In this talk, we study a class of anti-symmetric solutions to nonlinear equations with  $s$  order Laplacian,  $0 < s < 1$ . We prove that  $u=0$  is the unique solution in the often used defining space  $L_{\{2s\}}$ . Due to the anti-symmetric property of functions, it is surprising to see that the  $L_{\{2s\}}$  space for fractional Laplacian can be extended to  $L_{\{2s+1\}}$  space. In the extend space, we verify the existence of non-trivial solutions under some conditions.

#### 4. List of Participants

Name	Affiliation
程廷治	鲁东大学
郭千桥	西北工业大学
黄耿耿	复旦大学
雷雨田	南京师范大学
吕英姝	复旦大学
牛亚婷	复旦大学
王小龙	东华理工大学
张涛	烟台大学
周长亮	东华理工大学
来米加	上海交通大学
李从明	上海交通大学
李振杰	上海交通大学
廉媛媛	上海交通大学
王芳	上海交通大学
王邵东	上海交通大学
武乐云	上海交通大学
周春琴	上海交通大学
卓然	上海交通大学
梁警琪	上海交通大学
刘宸恺	上海交通大学
徐美清	上海交通大学
王丽丹	上海交通大学
周辉煌	上海交通大学

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